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Stochastic Geometry modeling and analysis of wireless networks (part II - applications)

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Outline for hour 1

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C.1 - Shot Noise Fields and Interference

Shot-Noise (1)

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- A shot-noise (SN) field is a non-negative random field $I_{\tilde{\Phi}}(y) \in \mathbb{R}$ defined $\forall\ y \in \mathbb{R}^d$ and is a functional of a marked point process $\tilde{\Phi}$.
- Response function $L : \mathbb{R}^d \times \mathbb{R}^d \times M \to \mathbb{R}$.

$$I_{\tilde{\Phi}}(y) = \sum_{(x_i, m_i) \in \tilde{\Phi}} L(y, x_i, m_i), \quad y \in \mathbb{R}^d.$$

Shot-Noise (2)

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- The expectation for the case of an i.m.PP is equal to

$$\mathbb{E}\left[I_{\tilde{\Phi}}(y)\right] = \int_{\mathbb{R}^d \times M} L(y, x, m)\, Q_x(dm)\, \Lambda(dx)\,.$$

using Campbell's formula, but can be infinite.

- If for each $y \in \mathbb{R}^d$ there exists ϵ_y s.t.

$$\int_{\mathbb{R}^d \times M} \left(\sup_{z \in \mathcal{B}_y(\epsilon_y)} L(y, x, m) \right) Q_x(dm)\, \Lambda(dx) < \infty$$

then with probability 1 the field is finite for all y.

Poisson Noise

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Suppose that $\tilde{\Phi}$ is an i.m. Poisson PP with intensity measure Λ and mark distribution $Q_x(dm)$. Then the Laplace transform is

$$\mathcal{L}_{I(y)}(s) \quad = \quad e^{-\int_{\mathbb{R}^d \times M}\left(1-e^{-sL(y,x,m)}\right)Q_x(dm)\Lambda(dx)}.$$

Interference as Shot-Noise (1)

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- Collection of transmitters distributed in space and sharing a common radio medium.
- Signal attenuation depends on distance and stochastic ingredients.
- The total power received from this collection of transmitters at a given location is a shot-noise field.

Interference as Shot-Noise (1)

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- Collection of transmitters distributed in space and sharing a common radio medium.
- Signal attenuation depends on distance and stochastic ingredients.
- The total power received from this collection of transmitters at a given location is a shot-noise field.

$$I(y) = I_{\tilde{\Phi}}(y) = \sum_{(x_i, p_i) \in \tilde{\Phi}} \frac{p_i F_i}{g(|y - x_i|)}, \quad y \in \mathbb{R}^2.$$

Interference as Shot-Noise (2)

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In the interference field:

- $\tilde{\Phi}$ is a stationary i.m.PP with points in $\mathbb{R}^2$ and intensity λ.
- The marks have some distribution $Q(t) = \mathbb{P}[F_i \leq t]$, independent of the point location.
- The function g depends only on the distance r, i.e.

$$\begin{aligned} g(r) &= (Ar)^{\alpha} \quad (case\ 1) \\ g(r) &= (1 + Ar)^{\alpha} \quad (case\ 2) \\ g(r) &= (A \max(r_0, r))^{\alpha} \quad (case\ 3) \end{aligned}$$

Interference as Shot-Noise (3)

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In the most general case for $\tilde{\Phi}$ and F

$$\mathbb{E}\left[I(y)\right] = p\mathbb{E}\left[F\right]2\pi\int_0^\infty \frac{1}{g\left(r\right)}\lambda r dr$$

$$\mathcal{L}_{I(y)}\left(s\right) = e^{-2\pi\lambda\int_0^\infty\left(1-\mathcal{L}_F\left(s\frac{1}{g(r)}\right)\right)rdr}$$

where $\mathcal{L}_F(s) = \int_0^\infty e^{-st}Q\left(dt\right)$.

Interference as Shot-Noise (4)

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In the special case for Poisson PP for $\tilde{\Phi}$ and exponential distribution for F

$$\mathbb{E}[I(y)] = 2\pi\lambda p \frac{1}{\mu} \int_0^\infty \frac{r}{g(r)} dr$$

$$\mathcal{L}_{I(y)}(s) = e^{-2\pi\lambda \int_0^\infty \frac{r}{1+\mu g(r)/s} dr}.$$

Interference as Shot-Noise (4)

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In the special case for Poisson PP for $\tilde{\Phi}$ and exponential distribution for F

$$\mathbb{E}\left[I(y)\right] = 2\pi\lambda p \frac{1}{\mu}\int_0^\infty \frac{r}{g(r)}dr$$

$$\mathcal{L}_{I(y)}(s) = e^{-2\pi\lambda\int_0^\infty \frac{r}{1+\mu g(r)/s}dr}.$$

Using the Case 1 for path-loss $(Ar)^\alpha$ we get

$$\mathcal{L}_I(s) = e^{-\frac{K(\alpha)}{A^2}\lambda\left(\frac{s}{\mu}\right)^{2/\beta}}, \qquad K(\alpha) = \frac{2\pi^2}{\alpha\sin(2\pi/\alpha)}.$$

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C.2 - Cellular Coverage (Andrews et al, 2011)

A random cellular network

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We have a cellular network with the following characteristics:

- Base stations (BSs) are modelled by a homogeneous Poisson PP Φ of intensity λ.
- The signal degradation follows the power loss model of (Case 1) with $\alpha > 2$.
- Assumptions: Rayleigh fading for the signal and general fading for the interference with density function f_g.
- The noise power is additive and constant with value σ^2.

A random cellular network (1)

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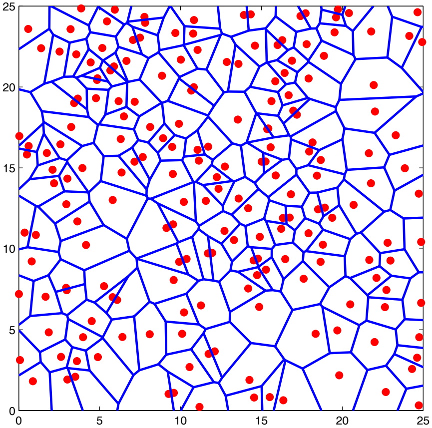

Fig. 1. Poisson distributed base stations and mobiles, with each mobile associated with the nearest BS. The cell boundaries are shown and form a Voronoi tessellation.

Typical User and the SINR

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- Each user is connected to the geographically nearest BS.
- The network performance is considered at the typical user
  Stationarity → Origin $(0,0)$.
- Typical user connected to b_o.
- The SINR is a r.v.

$$\mathrm{SINR} = \frac{hr^2}{\sigma^2 + \mathcal{I}_r},$$

where

$$\mathcal{I}_r = \sum_{i \in \Phi \setminus b_0} g_i R_i^{-\alpha}$$

is the cumulative interference from all the other base stations.

Coverage probability

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The coverage probability is defined as

$$p_c(T, \lambda, \alpha) = \mathbb{P}(\mathrm{SINR} > T).$$

The probability that a randomly chosen user achieves a target SINR T.

1st neighbour distribution

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- Lets denote r the random variable representing the distance from a typical user to his assigned nearest BS. We have

$$\begin{aligned}\mathbb{P}(r > s) &= \mathbb{P}(\text{ No BSs closer than s}) \\ &= e^{-\lambda\pi s^2} \\ &= 1 - F_r(s).\end{aligned}$$

F_r is the distribution function. Taking derivative with respect to s, we obtain the density function f_r,

$$f_r(s) = 2\pi\lambda s e^{-\lambda\pi s^2}.$$

Coverage Probability

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Theorem

The coverage probability of a typical mobile user is given by

$$p_c(T, \lambda, \alpha) = \pi\lambda \int_0^\infty e^{-\pi\lambda\nu\beta(T,\alpha) - \mu T \sigma^2 \nu^{\alpha/2}} d\nu,$$

where

$$\beta(T, \alpha) = \frac{2(\mu T)^{\frac{\alpha}{2}}}{\alpha} \mathbb{E}\left[g^{\frac{2}{\alpha}} (\Gamma(-2/\alpha, \mu T g) - \Gamma(-2/\alpha)) \right],$$

and $\Gamma(x)$ *et* $\Gamma(a, x)$ *denote the complete and incomplete gamma functions, respectively .*

Sketch of the proof

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$$p_c(T,\lambda,\alpha) = \int_{s>0} \mathbb{P}(SINR > T|r) f_r(s) ds$$
$$= \int_{s>0} \mathbb{P}(h > Tr^{\alpha}(\sigma^2 + \mathcal{I}_r)|r) f_r(s) ds$$

Due to the fact that $h \sim exp(\mu)$,

$$\mathbb{P}(h > Tr^{\alpha}(\sigma^2 + \mathcal{I}_r)|r) = \mathbb{E}[\mathbb{P}(h > Tr^{\alpha}(\sigma^2 + \mathcal{I}_r)|r, \mathcal{I}_r)]$$
$$= \mathbb{E}[e^{-\mu Tr^{\alpha}(\sigma^2 + \mathcal{I}_r)}|r, \mathcal{I}_r)]$$
$$= e^{-\mu Tr^{\alpha}\sigma^2} \mathcal{L}_{\mathcal{I}_r}(\mu r^{\alpha}),$$

where $\mathcal{L}_{\mathcal{I}_r}(s)$ is the Laplace transform of $\mathcal{I}_r$, conditioned on r.

Proof (2)

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$$
\begin{aligned}
\mathcal{L}_{\mathcal{I}_r}(s) &= \mathbb{E}[e^{-s\mathcal{I}_r}|r] \\
&= \mathbb{E}\left[e^{-s\sum_{i\in\Phi\setminus b_0} g_i R_i^{-\alpha}}\Big| r\right] \\
&= \mathbb{E}\left[\prod_{i\in\Phi\setminus b_0} e^{-sg_i R_i^{-\alpha}}\Big|\right] \\
&= \mathbb{E}\left[\prod_{i\in\Phi\setminus b_0} \mathbb{E}[e^{-sgR_i^{-\alpha}}]\Big| r\right] \\
&= e^{-2\pi\lambda\int_r^\infty (1-\mathbb{E}[e^{-sg\nu^{-\alpha}}])\nu d\nu} \\
&= e^{-2\pi\lambda\int_r^\infty\int_0^\infty (1-e^{-sx\nu^{-\alpha}}])f_g(x)dx\nu d\nu} \\
&= e^{-2\pi\lambda\int_r^\infty\int_0^\infty (1-e^{-sx\nu^{-\alpha}}])\nu f_g(x)d\nu dx}.
\end{aligned}
$$

First change of variable $v^{-\alpha} \to y$, substitution and second change of variable $r^2 \to v$.

Special cases

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- General fading, noise, $\alpha = 4$.

$$p_c(T,\lambda,4) = \frac{\pi^{\frac{3}{2}}\lambda}{\sqrt{\frac{T}{SNR}}} e^{\left(\frac{(\pi\lambda\beta(T,4))^2}{\frac{4T}{SNR}}\right)} Q\left(\frac{(\pi\lambda\beta(T,4))^2}{\sqrt{\frac{4T}{SNR}}}\right),$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-y^2/2}dy$.

- General fading, no noise, $\alpha > 2$

$$p_c(T,\lambda,4) = \frac{1}{\beta(T,\alpha)},$$

Evaluation case $\alpha = 4$, no noise

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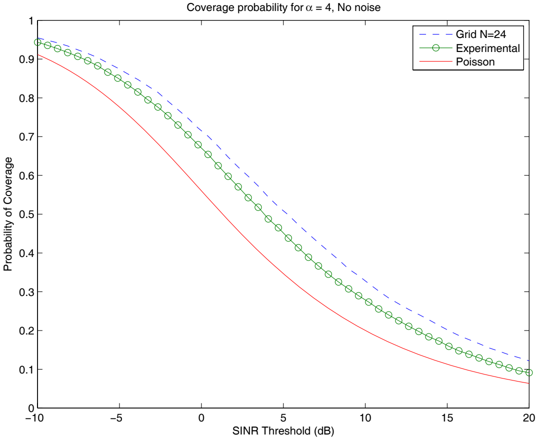

Special cases (2)

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- General fading, small but non zero noise, $\alpha > 2$

$$p_c(T,\lambda,4) = \frac{1}{\beta(T,\alpha)} - \frac{\mu T \sigma^2 (\lambda\pi)^{-\alpha/2}}{\beta(T,\alpha)} \Gamma\left(1+\frac{\alpha}{2}\right) + o(\sigma^2),$$

- Interference is Rayleigh fading

$$p_c(T,\lambda,\alpha) = \pi\lambda \int_0^\infty e^{-\pi\lambda\nu(1+\rho(T,\alpha)) - \mu T \sigma^2 \nu^{\alpha/2}} d\nu,$$

where $\rho(T,\alpha) = T^{\alpha/2} \int_{T^{-\alpha/2}}^\infty \frac{1}{1+u^{\alpha/2}} du$.

General Fading in the Signal

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If the probability density function of the signal fading is square integrable, the probability of coverage is given by

$$p_c(T,\lambda,\alpha) = \int_0^{\infty} 2\pi\lambda e^{-\pi\lambda s^2} \int_{-\infty}^{\infty} e^{-2\pi\sigma^2 i\nu} \mathcal{L}_{\mathcal{I}_r}(2\pi i\nu) \cdot \cdot \frac{\mathcal{L}_h(-2\pi(T\alpha)^{-1} i\nu) - 1}{2\pi i\nu} d\nu ds$$

where $\mathcal{L}_h$ is the Laplace transform of the signal fading.

Outline for hour 2

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D.1 - Cooperative Coverage (Giovanidis & Baccelli, 2013)

Cooperative Network Topology

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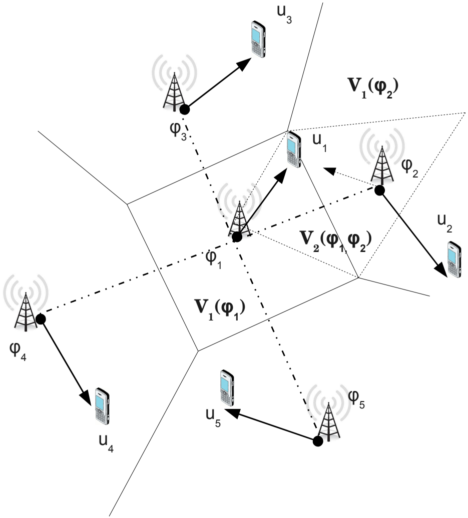

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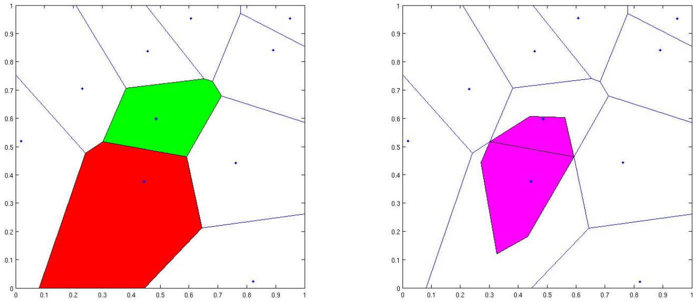
Voronoi Cells

- Base Stations located on atoms of a Poisson point process Φ.
- 1-Voronoi cell: defines the area of interest per Base Station.
- 2-Voronoi cell: locus of planar points closest to a pair of Base Stations.



Cooperation on the 2D-Plane

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- Each Base Station b_u is connected via a backhaul link of infinite capacity with all its Delaunay neighbours.
- Exactly 1 user u per 1-Voronoi cell randomly positioned.
- Each user is served by exactly 2 Base Stations:
    1. First closest geographic neighbour b_{u1}
    2. Second closest neighbour b_{u2}
- User u_i is primary user for b_{u1} and secondary user for b_{u2}.

General SINR

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- Consider user power $p_u = p = const$, for all users.

$$
\begin{aligned}
\mathrm{SINR}_u^{(\theta)}(\mathbf{a}, p) &= \frac{\mathcal{S}_u^{(\theta)}(a_u, p)}{\sigma^2 + \mathcal{I}_u^{(\theta)}(\mathbf{a}_{-u}, p)} \\
\mathcal{I}_u^{(\theta)}(\mathbf{a}_{-u}, p) &:= \sum_{v \neq u} \mathcal{S}_v^{(\theta)}(a_v, p) \\
\mathcal{S}_u^{(\theta)}(a_u, p) &:= h_{u1}(1 - a_u)p + h_{u2} a_u p + \\
&+ 2 a_u p \sqrt{h_{u1} h_{u2}} cos(\theta_{u1} - \theta_{u2}).
\end{aligned}
$$

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Coherent Transmission SINR

*En extra coherence term $2a_u p\sqrt{h_{u1}h_{u2}}cos\left(\theta_{u1}-\theta_{u2}\right)$ appears!

The term is maximized for the beneficial signal with transmission:

$$\theta_{u1} \quad = \quad \theta_{u2}.$$

For the interference part, its expected value is $\mathbb{E}\left[cos\left(\theta_{u1}-\theta_{u2}\right)\right]=0$, because the phases take values uniformly in $[0,2\pi]$.

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*Assuming phase knowledge for the two transmitting BSs and $a=1/2$

$$\mathrm{SINR}_u\left(1/2,p\right) = \frac{h_{u1}\frac{p}{2}+h_{u2}\frac{p}{2}+p\sqrt{h_{u1}h_{u2}}}{\sigma^2+\sum_{v\neq u}h_{v1}\frac{p}{2}+h_{v2}\frac{p}{2}}$$

Adaptive Geometric Policies

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- The beneficial user signal $\mathcal{S}_u$ is maximized either for $a_u = 0$ (No Coop) or for $a_u = 1/2$ (Full Coop).
- The choice depends on the ratio h_{u2}/h_{u1} and eventually r_{u1}/r_{u2} (fast fading neglected).

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We consider the family of geometric policies:

$$a^* = \begin{cases} 0 & \text{(No Coop)} \quad , \text{if } r_1 \leq \rho r_2 \\ \frac{1}{2} & \text{(Full Coop)} \quad , \text{if } r_1 > \rho r_2 \end{cases}$$

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- ρ: adaptive global parameter that separates the plane into Full Coop/No Coop zones.

Shape of the Cooperation Zones

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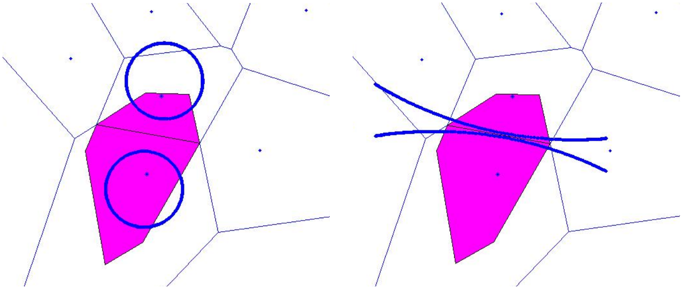
Given the locations of b_1 and b_2 on the plane, the geometric locus of points which satisfy $r_1 \leq \rho \cdot r_2$ for $\rho \in [0, 1]$ is a disc.

$\rho = 0$: Full Coop. everywhere, $\rho = 1$: No Coop. everywhere.



Coherent SINR

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$$
\begin{aligned}
\mathrm{SINR}(\rho, r_1, r_2) &= \frac{g_1 r_1^{-\beta}}{\sigma^2 + \mathcal{I}(\rho, r_2)} \cdot \mathbb{1}_{\{r_1 \leq \rho \cdot r_2\}} \\
&+ \frac{\frac{\left(\sqrt{g_1 r_1^{-\beta}} + \sqrt{g_2 r_2^{-\beta}}\right)^2}{2}}{\sigma^2 + \mathcal{I}(\rho, r_2)} \cdot \mathbb{1}_{\{r_1 > \rho r_2 \ \& \ r_1 \leq r_2\}}
\end{aligned}
$$

Coverage Probability

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- Consider a typical user u at the origin (Poisson p.p. property).
- Performance measure is the Coverage Probability

$$q_c\left(T,\lambda,\alpha,p,\rho\right) := \mathbb{P}_{\Phi}\left[\mathrm{SINR}\left(\alpha,p,r_1,r_2,\rho\right) > T\right].$$

1st & 2nd neighbour distribution

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- The joint p.d.f. of the distances (r_1, r_2) between u and its 1st and 2nd closest neighbour b_1 and b_2 equals

$$f_{r_1,r_2}(r_1, r_2) \quad = \quad (2\lambda\pi)^2 \, r_1 r_2 e^{-\lambda\pi r_2^2}.$$

1st & 2nd neighbour distribution

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- The joint p.d.f. of the distances (r_1, r_2) between u and its 1st and 2nd closest neighbour b_1 and b_2 equals

$$f_{r_1,r_2}(r_1, r_2) = (2\lambda\pi)^2 r_1 r_2 e^{-\lambda\pi r_2^2}.$$

- The probability of a user to lie on a No Coop. zone equals

$$\mathbb{P}[\text{No Coop}] = \mathbb{P}[r_1 \leq \rho r_2] = \rho^2 \in [0,1].$$

Channel Fading Distribution

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- The No Coop signal: $G_1 r_1^{-\beta}$.
  Has an exponential distribution.
- The Full Coop signal: $\frac{\left(\sqrt{G_1 r_1^{-\beta}}+\sqrt{G_2 r_2^{-\beta}}\right)^2}{2}$.
  The r.v. of the Full Coop signal $\frac{Z_{r_1,r_2}}{2}$ has a Laplace Transform, which can be calculated explicitly.

Channel Fading Distribution

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  The r.v. of the Full Coop signal $\frac{Z_{r_1,r_2}}{2}$ has a Laplace Transform, which can be calculated explicitly.
- Laplace Ordering: $X \leq_L Y$, if $\mathcal{L}_X(s) \geq \mathcal{L}_Y(s)$, $\forall s \geq 0$.

Lemma

The following Laplace-Stieltjes transform ordering inequality holds

$$G \leq_L \frac{Z_{r,r}}{2}.$$

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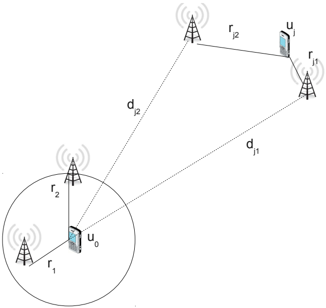
Geometry of Interference

- Signals for all users other than the typical one.
- Interference: shot noise field outside a ball of radius r_2.
- r_{j1}, r_{j2} distance of user u_j from its nearest two neighbours b_{j1}, b_{j2}.
  d_{j1}, d_{j2} distance of each of these two neighbours from the typical user.

$$\mathcal{I}(\rho, r_2) = \sum_{u_j \neq u_0} h_{j1} \mathbb{1}_{\{r_{j1} \leq \rho \cdot r_{j2}\}} + \frac{h_{j1} + h_{j2}}{2} \mathbb{1}_{\{r_{j2} \geq r_{j1} > \rho r_{j2}\}}.$$



Interference Marks per Base Station

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- Random user position in the cell → Bernoulli random variable

$$B_j \quad = \quad \begin{cases} 1 & \text{with prob. } \mathbb{P}\left[r_{j1} \leq \rho r_{j2}\right] = \rho^2 & \text{(No Coop)} \\ 0 & \text{with prob. } 1-\rho^2 & \text{(Full Coop)} \end{cases} \quad .$$

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- If $B_j = 1$: independent mark

$$\mathcal{M}_j := \mathcal{S}_j\left(0\right) = d_j^{-\beta} G_j.$$

Interference Marks per Base Station

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- If $B_j = 1$: independent mark

$$\mathcal{M}_j := \mathcal{S}_j(0) = d_j^{-\beta} G_j .$$

- If $B_j = 0$: independent mark (approximation $d_{j1} \approx d_{j2} = d_j$)

$$\mathcal{N}_j := \frac{h_{j1} + h_{j2}}{2} \approx d_j^{-\beta} \frac{(G_{j1} + G_{j2})}{2}$$

$G_j \sim \Gamma(1, p)$ and $\frac{(G_{j1}+G_{j2})}{2} \sim \Gamma(2, p/2)$, equal to p in expectation.

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Interference random variable

$$\begin{aligned}\mathcal{I}(\rho, r_2) &:= r_2^{-\alpha} G_2 B_2 + r_2^{-\alpha} \frac{G_1 + G_2}{2} (1 - B_2) \\ &+ \sum_{z_j \in \phi \setminus \{b_1, b_2\}} d_j^{-\alpha} G_j B_j + d_j^{-\alpha} \frac{G_{j1} + G_{j2}}{2} (1 - B_j)\end{aligned}$$

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Interference random variable

$$\mathcal{I}(\rho, r_2) := r_2^{-\alpha} G_2 B_2 + r_2^{-\alpha} \frac{G_1 + G_2}{2}(1 - B_2) + \sum_{z_j \in \phi \setminus \{b_1, b_2\}} d_j^{-\alpha} G_j B_j + d_j^{-\alpha} \frac{G_{j1} + G_{j2}}{2}(1 - B_j)$$

Theorem

The Laplace Transform of the Interference random variable for the model under study, with exponential fading channel power (Rayleigh fading), is equal to

$$\mathcal{L}_{\mathcal{I}}(s, \rho, r_2) = \mathcal{L}_{\mathcal{J}}(s, \rho, r_2) \cdot e^{-2\pi\lambda \int_{r_2}^{\infty} \left(1 - \mathcal{L}_{\mathcal{J}}(s, \rho, r)\right) r \, \mathrm{d}r},$$

where $\mathcal{L}_{\mathcal{J}}(s, \rho, r) = \rho^2 \frac{1}{1 + sr^{-\alpha} p} + \left(1 - \rho^2\right) \frac{1}{\left(1 + sr^{-\alpha} \frac{p}{2}\right)^2}$.

Its expected value equals

$$\mathbb{E}[\mathcal{I}(\rho, r_2, \alpha, p, \lambda)] = \frac{p}{(\alpha - 2) r_2^{\alpha}} \left(\alpha - 2 + 2\pi\lambda r_2^2\right).$$

and is independent of ρ.

2nd Neighbour Interference Elimination

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- The second neighbour interference is the most influential due to proximity (own primary user).
- Exchange such information over backhaul.
- Cancel it by Dirty Paper Coding (project beneficial signal on orthogonal space of interfering one).

$$\mathcal{I}_{DPC}(\rho, r_2) \quad := \quad \sum_{z_j \in \phi \setminus \{b_1, b_2\}} d_j^{-\alpha} G_j B_j + d_j^{-\alpha} \frac{G_{j1} + G_{j2}}{2} (1 - B_j)$$

General Coverage Probability

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Theorem

The coverage probability of a typical user for the cooperation scenario under study as a function of the parameter $\rho \in [0, 1]$ - given a fixed set of system values $\{T, \lambda, \beta, p\}$ - equals

$$
\begin{aligned}
q_c(\rho) &= q_{c,1}(\rho) + q_{c,2}(\rho) \\
&= \int_0^\infty \int_{\frac{r_1}{\rho}}^\infty (2\lambda\pi)^2 r_1 r_2 e^{-\lambda\pi r_2^2} \cdot e^{-\frac{r_1^\alpha}{p} T\sigma^2} \mathcal{L}_\mathcal{I}\left(\frac{r_1^\alpha}{p} T, \rho, r_2\right) \mathrm{d}r_2 \, \mathrm{d}r_1 \\
&+ \int_0^\infty \int_{r_1}^{\frac{r_1}{\rho}} (2\lambda\pi)^2 r_1 r_2 e^{-\lambda\pi r_2^2} \int_{-\infty}^\infty e^{-2i\pi\sigma^2 s} \mathcal{L}_\mathcal{I}(2i\pi s, \rho, r_2) \frac{\mathcal{L}_Z\left(-i\pi s/T, \frac{r_1^\alpha}{p}, \frac{r_2^\alpha}{p}\right) - 1}{2i\pi s} \mathrm{d}s \, \mathrm{d}r_2 \, \mathrm{d}r_1
\end{aligned}
$$

where $\mathcal{L}_\mathcal{I}(s, \rho, r_2)$ is the Laplace Transform of $\mathcal{I}$ and $\mathcal{L}_Z(s, \mu_1, \mu_2)$ is the Laplace Transform of the general fading r.v. Z_{r_1,r_2}.

Pros of the Scheme

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1. An extra coherent term appears at the beneficial signal

$$2a_i p\sqrt{h_{i1}h_{i2}}$$

2. Knowledge of 2nd closest neighbor exact position guarantees a ball of radius $r_2 > r_1$ interference free. $\mathbb{E}[r_2] = \frac{3}{4\sqrt{\lambda}}$.
3. Cooperation is in favor of cell-edge users.
4. An optimal choice $\rho^* \in [0,1]$ maximizes the coverage area.

Cons of the Scheme

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1. The 1st and 2nd closest neighbour to the user can also create first order interference. Very Severe!
2. The user may lie within the cooperation area even when $Z_{r_1,r_2} < G$ (The optimal choice of ρ^* should correct this).
3. There exist radii (r_1, r_2) for which the tail probability

$$\mathbb{P}[Z_{r_1,r_2} > T] < \mathbb{P}[G > T]$$

above some $T > 0$. Hence for high T Full Coop may not outperform No Coop.

Simulated Coverage and ρ dependence

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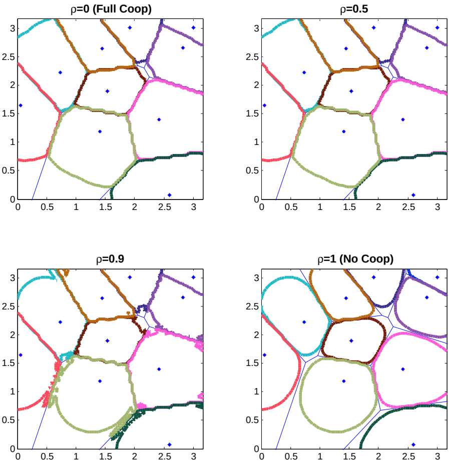

Coverage VS T ($\alpha = 4$)

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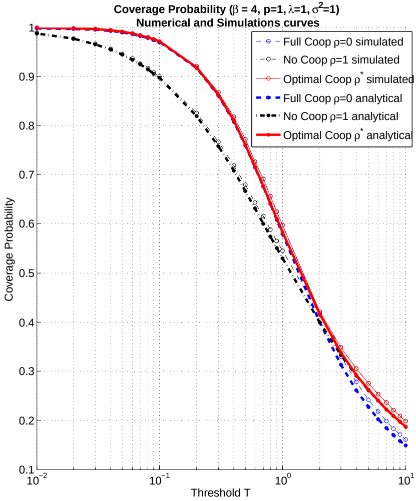

Optimal area ρ^*

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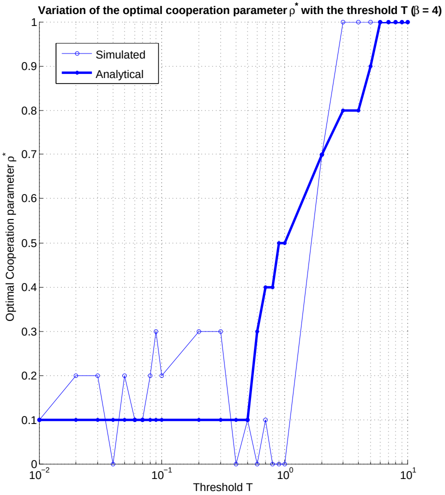

Coverage VS T ($\alpha = 4$) - DPC case

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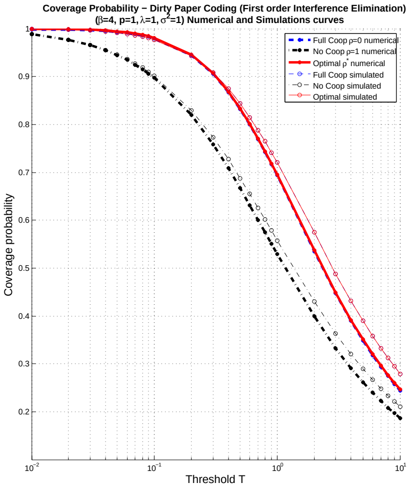

DPC Gains

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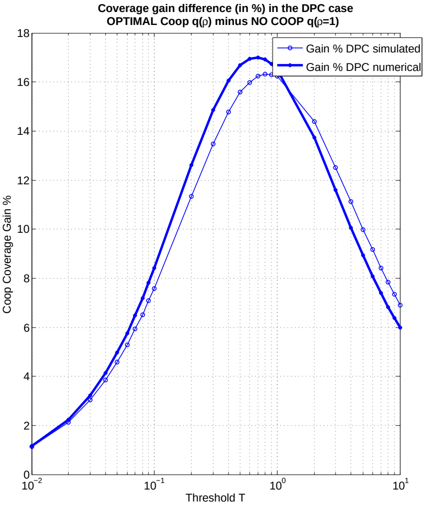

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END OF PART II