Game theoretic model for the downlink in cellular mobile networks: Nash equilibria and algorithmic convergence

A Game Theoretic Model

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The Problem

Setting: communication scenario

- set of base stations (BSs) M
- set of mobile stations (MSs) N
- channel quality between MS i and BS j: $h_{i,j}$
- noise level in the scenario σ^2

Aim

Determine good assignment of MSs to BSs and adjust power levels.

- $p_{i,j}$ is the power level which MS i receives from BS j, $\forall i \exists ! j : p_{i,j} > 0$.
- $\mathbf{p}$ is the vector of all $p_{i,j}$.
- $p_{i,j} > 0 \iff \alpha_i = j$.

SINR

To be optimized: The Signal-to-interference-and-noise-ratio γ_i:

Definition

$$\gamma_i(\mathbf{p}) = \frac{h_{i,\alpha_i} p_{i,\alpha_i}}{\mathcal{I}_i + \sigma^2}$$

Interference:

Definition

$$\mathcal{I}_i = \sum_{k \in I_i} h_{i,\alpha_k} p_{k,\alpha_k}$$

I_i is set of mobiles, the signals of which interfere with i.

Limited power resources

- Limited power resources of BSs: cost per power unit demanded by MS.
- c_j non-negative, convex and strictly increasing pricing function.
- The higher the load, the higher the cost per power unit: $p_{i,\alpha_i} \cdot c_{\alpha_i}(\ell_{\alpha_i}(\mathbf{p}))$, where $\ell_j(\mathbf{p}) = \sum_{i \in N} p_{i,j}$ the load on BS j

Utility function

Definition (Utility function)

$$u_i(\mathbf{p}) = \log(1 + (\gamma_i(\mathbf{p}))) - p_{i,\alpha_i} \cdot c_{\alpha_i}(\ell_{\alpha_i}(\mathbf{p})),$$

Non-linear program

$$
\begin{aligned}
\textbf{(LBBA)} \quad & \max_{\mathbf{p}} \sum_{i \in N} u_i(\mathbf{p}) & & \\
& \text{s.t.} \sum_{j \in M} x_{i,j} = 1 & \forall i \in N & \quad \text{(ASG)} \\
& \gamma_i(\mathbf{p}) \geq \text{threshold} & \forall i \in N & \quad \text{(THR)} \\
& p_{i,j} \leq M x_{i,j} & \forall i \in N, j \in M & \quad \text{(COUP1)} \\
& x_{i,j} \leq \varepsilon p_{i,j} & \forall i \in N, j \in M & \quad \text{(COUP2)} \\
& p_{i,j} \geq 0 & \forall i \in N, j \in M & \quad \text{(DOM1)} \\
& x_{i,j} \in \{0, 1\} & \forall i \in N, j \in M & \quad \text{(DOM2)}
\end{aligned}
$$

Game theoretic approach

- MSs as rational, non-cooperative players
- Step by step, players select a BS and a favorable power level
- At all times, players need information about their channel values, noise as well as other players' strategies
- When PNE is reached, the BS allocation and power management is decided

Question

Is there a PNE in the game?

2-Player Games

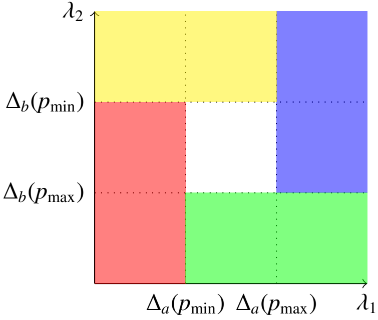

Figure : $\Delta_a(p_{\min}) < \Delta_a(p_{\max}), \Delta_b(p_{\min}) > \Delta_b(p_{\max})$. PNE does not always exist, but if so it is unique.

2-Player Games

In two player games with two BSs, where all MSs contribute to interference:
We cannot guarantee a PNE!

Restriction of the Problem

No inter-cell interference!

Similarity to Problem in the Uplink

Neglecting the cost function, Perlaza et al. (2009) have examined the problem for the uplink.
They found the game to be an exact potential game $\implies$ PNE.

No Exact Potential found!

SINR in the downlink:

$$\gamma_i(\mathbf{p}) = \frac{h_{i,\alpha_i} p_{i,\alpha_i}}{\sum_{\{k \in N : \alpha_k = \alpha_i\} \setminus \{i\}} h_{i,\alpha_i} p_{k,\alpha_i} + \sigma^2}$$

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Further Restriction: Assignment Problem

All MSs i assigned to BS j receive $p_{i,j} = p_j$.

Congestion Game with Player-Specific Payoff Function

Now, if player i chooses BS j, the SINR for player i only depends on the number of players having chosen j, so:

$$u_{i,j}(y) = g_i\left(\frac{1}{y - 1 + s_{i,j}}\right) - c_j(p_j y) \cdot p_j.$$

Remember definition of utility function:

$$u_i(\mathbf{p}) = g_i\Big(\frac{h_{i,\alpha_i} p_{i,\alpha_i}}{\sum_{\{k \in N : \alpha_k = \alpha_i\} \setminus \{i\}} h_{i,\alpha_i} p_{k,\alpha_i} + \sigma^2}\Big) - p_{i,\alpha_i} \cdot c_{\alpha_i}(\ell_{\alpha_i}(\mathbf{p})).$$

Congestion Game with Player-Specific Payoff Function

All $u_{i,j}$ are monotonically non-increasing $\implies$ existence of PNE and game is weakly acyclic.
Therefore we can find a probabilistic algorithm to achieve a PNE.

Weighted Congestion Game with Player-Specific Payoff Function

If we accept different energy levels depending on the MS/BS pair, then we find an instance without PNE.

Price of Anarchy

Price of Anarchy?

Thank you

Thank you for your attention!